B.Sc. Part II (Hons.) 4th Paper

DIFF. EQUATIONS

Linear Diff. Eqns with constant coefficients (contd.)

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

Here, consider $R = e^{ax}$.

Q. $(D^2+5D+6)y = e^{2x}$

Soln: For CF, $D^2+5D+6=0$

$\Rightarrow (D+3)(D+2)=0 \Rightarrow D=-3,-2$

$\therefore$ C.F. $= c_1 e^{-3x} + c_2 e^{-2x}$

Now, $PI = \dfrac{1}{D^2+5D+6} e^{2x}$

$= e^{2x} \times \dfrac{1}{2^2+5\times 2+6}$

[Note: $e^{2x} = e^{ax}$ i.e. $a=2$. Put $D=a$ if the denominator does not equal to zero.]

$\Rightarrow PI = \dfrac{e^{2x}}{20}$.

Hence, complete soln $= CF+PI$

$\Rightarrow y = c_1 e^{-3x} + c_2 e^{-2x} + \dfrac{e^{2x}}{20}$

2. Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$

Soln. $\Rightarrow$ The given equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$$

$$\Rightarrow D^2y - 5Dy + 6y = e^{4x}$$

$$\Rightarrow (D^2 - 5D + 6)y = e^{4x}$$

For C.F. $D^2 - 5D + 6 = 0$

$$\Rightarrow D^2 - 3D - 2D + 6 = 0$$

$$\Rightarrow D(D-3) - 2(D-3) = 0 \Rightarrow (D-3)(D-2) = 0$$

$$\Rightarrow D = 3, 2.$$

$$\therefore CF = c_1 e^{3x} + c_2 e^{2x}.$$

Now, for P.I. $PI = \frac{1}{(D^2 - 5D + 6)} e^{4x}$

$$\Rightarrow PI = e^{4x} \times \frac{1}{4^2 - 5 \times 4 + 6} = \frac{e^{4x}}{2}.$$

Hence, the complete soln is given by

$$y = CF + PI$$

$$\Rightarrow y = c_1 e^{3x} + c_2 e^{2x} + \frac{1}{2} e^{4x}$$

Q. Solve $(D^2-4D+1)y = e^{2x} - e^{-x}$.

Soln. The given equation

$$(D^2-4D+1)y = e^{2x} - e^{-x}.$$

CF is given by,

$$D^2-4D+1=0$$

$$\Rightarrow D^2-4D+4=3 \Rightarrow (D-2)^2 = 3$$

$$\Rightarrow D-2 = \pm\sqrt{3} \Rightarrow D = 2\pm\sqrt{3}.$$

$$\therefore CF = c_1 e^{(2+\sqrt{3})x} + c_2 e^{(2-\sqrt{3})x}$$

$$\Rightarrow CF = e^{2x}\left[c_1 e^{\sqrt{3}x} + c_2 e^{-\sqrt{3}x}\right] \quad \text{—(1)}$$

Now, we find P.I.

$$PI = \frac{1}{(D^2-4D+1)}\left(e^{2x} - e^{-x}\right)$$

$$\Rightarrow PI = \frac{1}{D^2-4D+1}e^{2x} - \frac{1}{D^2-4D+1}e^{-x}$$

$$= e^{2x} \times \frac{1}{2^2-4\times2+1} - e^{-x} \times \frac{1}{(-1)^2+4\times1+1}$$

$$\Rightarrow PI = -\frac{1}{3}e^{2x} - \frac{1}{6}e^{-x}. \quad \text{—(2)}$$

Hence, complete soln is given by $y = CF + PI$ where CF & PI are given by (1),(2) respectively

$\boxed{Q.}$ Solve $(D^2+1)y = (e^x+1)^2$

Soln The given equation

$$D^2+1=0 \Rightarrow D = \pm i.$$

$$\Rightarrow CF = c_1 e^{ix} + c_2 e^{-ix}$$

$$= c_1(\cos x + i\sin x) + c_2(\cos x - i\sin x)$$

$$= (c_1+c_2)\cos x + i(c_1-c_2)\sin x$$

$$\Rightarrow CF = A\cos x + B\sin x,$$

where $A = c_1+c_2$, $B = i(c_1-c_2)$.

Now, $$PI = \frac{1}{(D^2+1)}(e^x+1)^2$$

$$\Rightarrow PI = \frac{1}{D^2+1}(e^{2x}+2e^x+1)$$

write $1 = e^{0x}$

$$\Rightarrow PI = \frac{1}{D^2+1}e^{2x} + \frac{2}{D^2+1}e^x + \frac{1}{D^2+1}\cdot 1$$

$$\Rightarrow PI = e^{2x}\times\frac{1}{2^2+1} + e^x\times\frac{2}{1^2+1} + \frac{1}{0^2+1}$$

$$\Rightarrow PI = \frac{e^{2x}}{5} + e^{2x} + 1.$$

$\therefore$ complete soln $= CF + PI$

$$\Rightarrow y = A\cos x + B\sin x + \frac{e^{2x}}{5} + e^{2x} + 1$$